HINTS: PROBLEM SET 5

- §1.8, # 7.
  - To check that a given vector is tangent to S^n at a point x, consider $S^n = f^{-1}(1)$, $f : \mathbb{R}^{n+1} \to \mathbb{R}$, $f(x) = |x|^2$. Then use the Proposition on the bottom of page 24.
  - If $x = (x_1, x_2, \ldots, x_n, x_{n+1}) \in S^n$ for n odd, use a similar construction to that for S^1. Why is it important that $n+1$ is even?
- §1.8, # 8.
  - Recall that if v is a nonzero vector, then $v/|v|$ is a unit vector. (Is the vector you constructed in the previous problem already a unit vector anyway?)
  - Let v and w be two perpendicular unit vectors in $\mathbb{R}^{n+1}$. Think of w as a point in S^n, and v as a tangent vector in $T_w S^n$. For any angle θ, show that

    $$w_\theta = w \cos\theta + v \sin\theta$$

    is the unit vector obtained from w by rotating at an angle θ in the direction of v.
  - What happens when $\theta = \pi$?
  - Construct the required homotopy given the rotation formula above.
- §1.8, # 10.
  - Start from the Whitney embedding theorem: every k-dimensional manifold can be embedded in $\mathbb{R}^{2k+1}$.
  - Slightly modify the proof on page 51. In particular, of the two maps h and g, which is relevant in this case?
- §2.2, # 4.
  - Is there a smooth map from $\mathbb{R}^n$ to itself which has no fixed points?
- §2.3, # 4.
  - In the discussion in the middle of page 69, what happens if the map f is an inclusion of X into $\mathbb{R}^N$?